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Charlottesville, Virginia 22901

DEVELOPMENT OF FINITE ELEMENT MODELS FOR THE EARTH'S GRAVITY FIELD

PHASE II: FINE STRUCTURE DISTURBANCE GRAVITY REPRESENTATIONS

Submitted to:

U. S. Army Engineer Topographic Laboratories Fort Belvoir, Virginia 22060

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March 1977

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Final Report

DEVELOPMENT OF FINITE ELEMENT MODELS FOR THE EARTH'S GRAVITY FIELD

PHASE II: FINE STRUCTURE DISTURBANCE GRAVITY REPRESENTATIONS

Contract No. DAAG 53-76-C-0067

Submitted to:

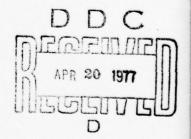
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## 1.0 SUMMARY

This research and development effort has obtained the following results:

- Software suitable for the construction of finite element gravity disturbance fields has been developed.
- Software suitable for calculation of the gravity disturbance model within finite element fields has been developed.
- Example finite element gravity disturbance fields have been developed and tested.

#### 2.0 PREFACE

This report constitutes the final report of Phase II of Contract No. DAAG-53-76-C-0067 performed by the University of Virginia for the U. S. Army Engineer Topographic Laboratories, Fort Belvoir, Virginia, under the sponsorship of the Defense Mapping Agency. The Phase II effort is concerned with representing local fine structure gravity by locally valid (finite element) gravity functions. The Phase I effort, documented in Reference (1), deals with replacing globally valid gravity representations (such as spherical harmonic series) by an equivalent family of locally valid gravity functions. The Phase I effort yielded a macro gravity model suitable for high speed satellite orbit integration. The Phase II effort (documented herein) yielded fine structure gravity representations for efficient disturbance acceleration calculation (to use, for example, in inertial guidance equations).

The authors appreciate the technical inputs of Mr. L. A. Gambino (technical monitor), Dr. R. W. Ballew, Mr. H. W. Howard, and Mr. B. L. Decker.

#### 3.0 INTRODUCTION

The finite element concept may be considered as an extension to three dimensions of one dimensional piecewise approximation techniques. Some quantity that is functionally related in a "complicated" fashion to position over a "large" interval may be replaced for much "smaller" intervals of position with much "less complicated" approximating functions. Thus the complicated function, which will normally be "more expensive" to evaluate, will have been replaced by a set of locally valid functions which will normally be "less expensive" to evaluate.

The idea may easily be extended to three dimensions. A quantity which is functionally related to position over some large region may be replaced by a set of approximations which are valid only in small volumes of the large region, but which are easier (faster computationally) to evaluate. The finite element concept may be applied to any of a variety of modeling problems; this investigation centered specifically on fine structure gravity modeling.

Previously, earth-fixed spherical coordinate based finite element fields have been successfully used to replace globally valid spherical harmonic representations of the geopotential and its derivatives. But because the model used to simulate gravity disturbance data (Model 310) consists of a set of point masses placed in relation to the Geodetic Reference Surface of 1967, geodetic coordinates (H,  $\lambda$ ,  $\phi$  - see Figure 1) were used in the finite element modeling process.

Finite element fields may be defined with geodetic coordinates as shown in Figure 2. The region to be modeled is defined by a set of maximum bounds ( $H_{max}$ ,  $\lambda_{max}$ ,  $\phi_{max}$ ) and a set of minimum bounds ( $H_{min}$ ,  $\lambda_{min}$ ,  $\phi_{min}$ ). The region is then divided into a set of smaller volumes, or finite element cells, where each cell has dimensions  $H_{cell}$ ,  $\phi_{cell}$ . Thus the 300 km × 10° × 10° region above the reference ellipsoid,

altitude 0 - 300 km,

longitude 70°E - 80°E, and

latitude 25°S - 35°S,

could be modeled by one hundred (100) cells each 300 km  $\times$  1°  $\times$  1° or four (4) cells each 300 km  $\times$  5°  $\times$  5° or one cell 300 km  $\times$  10°  $\times$  10°, etc.

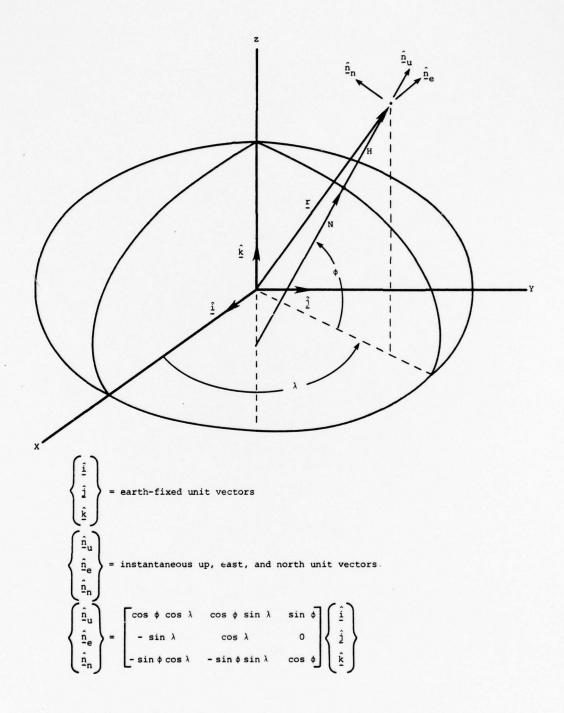


Figure 1 Earth-Fixed Rectangular and Geodetic Coordinate Systems.

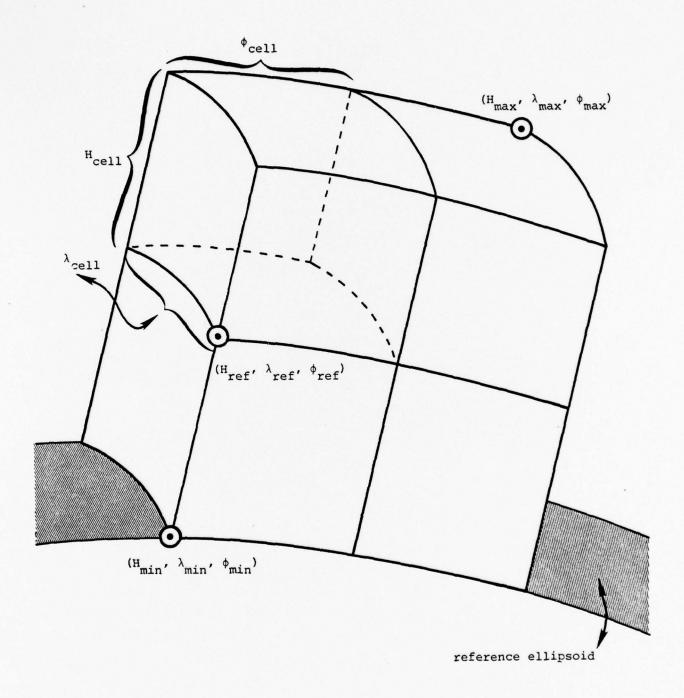


Figure 2 Geodetic Finite Element Field.

The approximation valid in any one cell is generated independentally of the approximation for any other cell. A set of gravity disturbance "observations" are produced for each cell directly from Model 310 and then approximated for only that cell in a least squares fitting process. The approximating models thus generated are then stored on a high speed rotating mass storage device for later use in the computation of the approximate gravity disturbance in any finite element cell of the region modeled.

### 4.0 FINITE ELEMENT FORMULATION

Define the gravity disturbance,  $\delta g$ :

$$\frac{g}{\text{actual}} = \frac{g}{\text{reference}} + \delta g$$
 (1)

$$\delta \underline{g} = \begin{cases} \delta g_{up} \\ \delta g_{east} \\ \delta g_{north} \end{cases} = \underline{g}_{actual} - \underline{g}_{reference}$$
 (2)

Now, replace the components of  $\delta g$  with locally valid Chebyshev polynomial approximations,  $\delta G$ , of the form,

$$\delta \underline{G} = \begin{pmatrix} G_{up} \\ G_{east} \\ G_{north} \end{pmatrix} = \sum_{n=0}^{N} \sum_{i=0}^{n-i} \sum_{j=0}^{C_{up}} \begin{pmatrix} C_{up_{ijk}} \\ C_{east_{ijk}} \\ C_{north_{ijk}} \end{pmatrix} \underline{T}_{i} (X_{1}) \underline{T}_{j} (X_{2}) \underline{T}_{k} (X_{3})$$
(3)

(4)

where,

N = NORDER = the highest order Chebyshev polynomials,

$$k = n - i - j$$

$$X_1 = (H - H_{ref})/H_{cell}$$

$$x_2 = (\lambda - \lambda_{ref})/\lambda_{cell}$$

$$x_3 = (\phi - \phi_{ref})/\phi_{cell}$$

 $T_n(X)$  = shifted Chebyshev polynomials, and

 $C_{up_{ijk}}$ ,  $C_{east_{ijk}}$ , and  $C_{north_{ijk}}$  are constant coefficients determined via least square fits so that  $\sum_{i=1}^{m} (\delta G_{up} - \delta g_{up})_{i}^{2}, \sum_{i=1}^{m} (\delta G_{east} - \delta g_{east})_{i}^{2},$ 

and  $\sum_{i=1}^{m} (\delta G_{north} - \delta g_{north})_{i}^{2}$  are minimized for some local volume of space

(i.e. the cell being modeled).

Chebyshev polynomials,  $t_n(X)$ , of order n may be generated:

n	t <sub>n</sub> (X)	
0	1	
1	х	
2	$2x^2 - 1$	

For  $n \ge 2$ , a recursion may be used:

$$t_n(x) = 2x t_{n-1}(x) - t_{n-2}(x), -1 \le x \le 1$$
 (5)

Shifted Chebyshev polynomials,  $T_n(\bar{X})$  may be computed by substituting

$$X = 2\bar{X} - 1$$

so that,

$$T_n(\bar{X}) = t_n(X) = t_n(2\bar{X} - 1), \qquad 0 \le \bar{X} \le 1$$
 (6)

To generate shifted Chebyshev polynomials a set of X's with values between 0 and 1 are needed. The difference between some point  $(H,\lambda,\phi)$  in a cell and a reference point  $(H_{\text{ref}},\lambda_{\text{ref}},\phi_{\text{ref}})$  (the "lowermost corner" of the cell) is divided by the cell's dimensions,  $H_{\text{cell}},\lambda_{\text{cell}},\phi_{\text{cell}}$ , as in (4) to obtain a set of non-dimensional coordinates,  $X_1$ ,  $X_2$ ,  $X_3$ , for each cell. The non-dimensional coordinates may then be used to generate the shifted Chebyshev polynomials.

To determine the three sets of constant coefficients,  $C_{\rm up}$ ,  $C_{\rm east}$ , and  $C_{\rm north}$ , for each cell, formulate and solve the linear least squares problems,

$$||\delta G_{up} - \delta g_{up}|| = ||AC_{up} - \delta g_{up}|| = \min.$$

$$||\delta G_{east} - \delta g_{east}|| = ||AC_{east} - \delta g_{east}|| = \min.$$

$$||\delta G_{north} - \delta g_{north}|| = ||AC_{north} - \delta g_{north}|| = \min.$$
(7)

where the coefficient matrix, A, has the form,

$$A = \begin{bmatrix} 1 & T_{1}(X_{31}) & T_{1}(X_{21}) & T_{1}(X_{11}) & T_{2}(X_{31}) & T_{1}(X_{31})T_{1}(X_{21}) \cdots \\ 1 & T_{1}(X_{32}) & T_{1}(X_{22}) & T_{1}(X_{12}) & T_{2}(X_{32}) & T_{1}(X_{32})T_{1}(X_{22}) \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & T_{1}(X_{3m}) & T_{1}(X_{2m}) & T_{1}(X_{1m}) & T_{2}(X_{3m}) & T_{1}(X_{3m})T_{1}(X_{2m}) \cdots \end{bmatrix}$$
(8)

and the three coefficient vectors, C up' C east' and C north' have the form

the three coefficient vectors, 
$$C_{up}$$
,  $C_{east}$ , and  $C_{north}$ , have the form 
$$\begin{bmatrix} C_{up}_{000} \\ C_{up}_{001} \\ C_{up}_{010} \\ C_{up}_{100} \\ C_{up}_{002} \\ C_{up}_{011} \\ \vdots \\ C_{up}_{N00} \end{bmatrix}, C_{east} = \begin{bmatrix} C_{east}_{000} \\ C_{east}_{001} \\ C_{east}_{100} \\ C_{east}_{002} \\ C_{east}_{011} \\ \vdots \\ C_{east}_{N00} \end{bmatrix}, C_{north} = \begin{bmatrix} C_{north}_{000} \\ C_{north}_{001} \\ C_{north}_{100} \\ C_{north}_{002} \\ C_{north}_{011} \\ \vdots \\ C_{north}_{N00} \end{bmatrix}$$
The state of the form of the proof of t

where

N = NORDER,

and the three sets of gravity disturbance observations,  $\delta g_{up}$ ,  $\delta g_{east}$ , and δg<sub>north</sub>, are

$$\delta g_{\rm up} = \left\{ \begin{array}{l} \delta g_{\rm up}(x_{1_1}, x_{2_1}, x_{3_1}) \\ \delta g_{\rm up}(x_{1_2}, x_{2_2}, x_{3_2}) \\ \delta g_{\rm up}(x_{1_3}, x_{2_3}, x_{3_3}) \\ \vdots \\ \delta g_{\rm up}(x_{1_m}, x_{2_m}, x_{3_m}) \end{array} \right\} \delta g_{\rm east} = \left\{ \begin{array}{l} \delta g_{\rm east}(x_{1_1}, x_{2_1}, x_{3_1}) \\ \delta g_{\rm east}(x_{1_2}, x_{2_2}, x_{3_2}) \\ \delta g_{\rm east}(x_{1_3}, x_{2_3}, x_{3_3}) \\ \vdots \\ \delta g_{\rm east}(x_{1_m}, x_{2_m}, x_{3_m}) \end{array} \right\}$$

$$\delta g_{\text{north}} = \begin{cases} \delta g_{\text{north}}(x_{1_{1}}, x_{2_{1}}, x_{3_{1}}) \\ \delta g_{\text{north}}(x_{1_{2}}, x_{2_{2}}, x_{3_{2}}) \\ \delta g_{\text{north}}(x_{1_{3}}, x_{2_{3}}, x_{3_{3}}) \\ \vdots \\ \delta g_{\text{north}}(x_{1_{m}}, x_{2_{m}}, x_{3_{m}}) \end{cases}$$
(10)

where  $\{(X_1, X_2, X_3), i = 1, 2, ..., m\}$  = a specified set of  $(H_i, \lambda_i, \phi_i)$  coordinates - usually a uniform observation grid in any one cell. The gravity disturbance observations are determined directly from Model 310 evaluations on the observation grid in each cell.

## 4.1 Discussion

Certain aspects of the process by which finite element fields are created may be capitalized on under special circumstances. When it is known beforehand that uniformly gridded gravity disturbance observations will be available, only one least squares matrix (A matrix) need be generated. This is true because the set of non-dimensional coordinates,  $\{(x_1, x_2, x_3), i=1, 2, \ldots, m\} \text{ of one cell's observation grid will be the same as every other cell's non-dimensional observation grid coordinates when the observation grid pattern is set for all cells. If the same number of observations are made in each cell and the positions of the observations points are the same relative to the reference point (Href' ref') in each cell, then all cells will have the same A matrix. The A matrix, after all, consists only of products of Chebyshev polynomials, which are functions only of the non-dimensional coordinates of the observation points.$ 

This all means that only one A-matrix need be generated for the first cell of a finite element field; but the gravity observations must be generated for each cell. A reduction of the A-matrix to upper triangular form may be performed for the least squares fitting process, after which each set of observations may be similarly reduced and then back-substituted to produce a set of coefficients for each cell. The A-matrix, again, needs to be reduced only once.

It should be noted that this method of finite element field generation does not rely on Model 310 or any other particular gravity disturbance model. Model 310 could be replaced by any other process that is capable of producing the gravity disturbance observations at the observation grid points of each cell.

Once a finite element field has been created, the Chebyshev polynomial gravity disturbance approximation  $\delta G$  may be computed by evaluating (3).

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The subscripts ijk always follow the same pattern for a given NORDER. This means that a "current coefficient number",  $\ell$ , may be attached to each group of subscripts ijk and that the total number of coefficients, NC, necessary to model any one component of the gravity disturbance in one cell will be constant for a given NORDER. Table 1 shows these relationships for various NORDER's.

Since we can precompute and save the subscripts, ijk, in the appropriate pattern such that they are functions of only the current coefficient number,  $\ell$ , (3) can be written,

$$\delta \underline{G} = \begin{pmatrix} \delta G_{up} \\ \delta G_{east} \\ \delta G_{north} \end{pmatrix} = \sum_{\ell=1}^{NC} \begin{pmatrix} C_{up} \\ C_{east} \\ C_{east} \\ C_{north} \end{pmatrix} T_{i} (1) T_{j} (X_{2}) T_{k} (X_{3})$$
 (11)

where,

NC = the total number of coefficients for one component of the gravity disturbance, as above

ijk = precomputed and saved as functions of  $\ell$ 

 $X_1, X_2, X_3$  = functions of H, $\lambda$ , $\phi$  and the constants H $_{\rm ref}$ ,  $\lambda_{\rm ref}$ ,  $\phi_{\rm ref}$ ,  $\phi_{\rm ref}$ ,  $\phi_{\rm cell}$ ,  $\phi_{\rm cell}$  as in (4) above. The total number of summations in (11) is the same as in (3) but the amount of bookkeeping has been cut down tremendously by doing business in this fashion. Computational savings of up to 50% were obtained by using (11) in lieu of (3) for evaluation purposes. Table 1 was also used in the generation of the A matrix, but the saving here is not as significant due to the previously discussed fact that the A matrix is only generated once.

NORDER	<u>ijk</u>	<u>L</u>	NC
0	0 0 0	1	1
1	0 0 1	2	
	0 1 0	3	
	1 0 0	4	4
2	0 0 2	5	
	0 1 1	6	
	0 2 0	7	
	1 0 1	8	
	1 1 0	9	
	2 0 0	10	10
3	0 0 3	11	
	0 1 2	12	
	:::	:	
	3 0 0	20	20
•		•	•
:	:::	•	
N	0 0 N	•	
	0 l N-1		
	:::		,
	N O O	NC	$\sum_{n=1}^{N+1} \left( \begin{array}{c} n \\ \sum m \\ m=1 \end{array} \right)$

Table 1 Subscript Patterns for Various NORDER's.

#### 5.0 DISCUSSION OF SOFTWARE

A variety of FORTRAN programs and subroutines were written during the course of this research effort. The creation of finite element gravity disturbance fields may be accomplished by the execution of PROGRAM LOCALG followed by the execution of PROGRAM FINEG. Once a finite element field has been generated, the gravity disturbance approximation may be calculated at any point in the finite element field by SUBROUTINE FINITE. To calculate the gravity disturbance from the mass points of Model 310, SUBROUTINE PTMASS was written. SUBROUTINE PTMASS was used both to produce observations for the least squares fitting process and to produce observations to test against for error analyses of finite element fields. Because SUBROUTINE PTMASS required the precomputed products of each point mass and the gravitational constant and the earth-fixed Cartesian coordinates of each point mass of Model 310, another small program, PROGRAM MASPOS, was written to calculate and store these quantities.

PROGRAM LOCALG requires input from the user which defines the finite element field to be generated. It writes the coefficient matrix (A matrix) and a set of gravity disturbance observations for each cell of the finite element field onto a magnetic tape (TAPE 3 - this may be any sequential mass storage device) for later use by PROGRAM FINEG. PROGRAM LOCALG is the first part of the local gravity disturbance modeling process. Figure 3 shows the logic flow for PROGRAM LOCALG.

The second part of the local gravity disturbance modeling process the reduction of the A matrix and the calculation of the fine structure
gravity model coefficients (9) - is accomplished by PROGRAM FINEG.

PROGRAM FINEG reads the A matrix and observation from the magnetic tape
(really, sequential file) created by PROGRAM LOCALG. PROGRAM FINEG
reduces the A matrix and calculates each cell's model coefficients in a
least squares reduction. The model coefficients are written onto a
random access file (preferably a high speed disk file but any high speed
mass storage random access file could be used). The logic flow for
PROGRAM FINEG is shown in Figure 4.

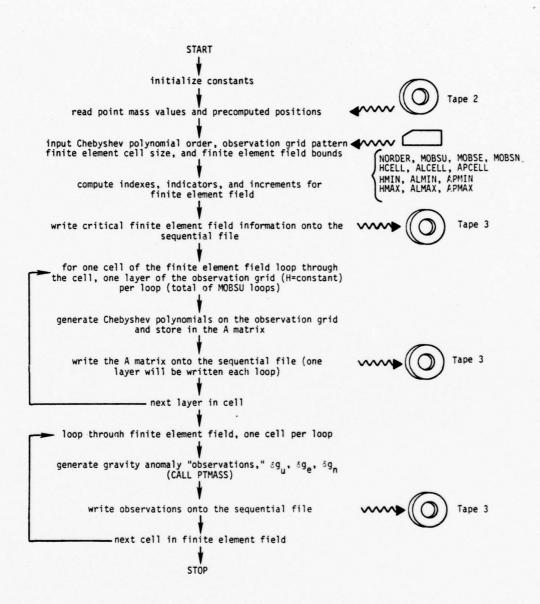


Figure 3 PROGRAM LOCALG Logic Flow.

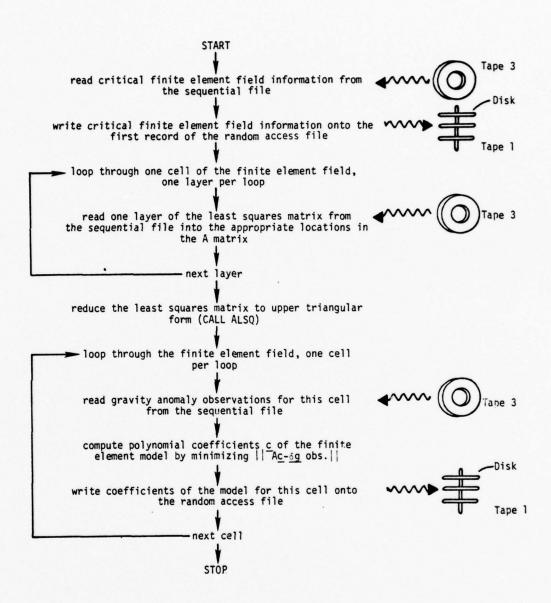


Figure 4 PROGRAM FINEG Logic Flow.

SUBROUTINE FINITE evaluates (11) above at some point  $(H,\lambda,\phi)$ . Using the sets of finite element coefficients previously generated by PROGRAM FINEG, SUBROUTINE FINITE returns an evaluation of the gravity disturbance approximation (11) at the specified point. The logic flow for SUBROUTINE FINITE is shown in Figure 5.

Gravity disturbance observations were produced for this study by SUBROUTINE PTMASS, which uses the 1080 point masses of Model 310 and (12) below to compute the gravity anomaly,  $\delta \underline{q}$ , at some point P(x,y,z):

$$\delta \underline{g} = \begin{pmatrix} \delta g_{\mathbf{x}} \\ \delta g_{\mathbf{y}} \\ \delta g_{\mathbf{z}} \end{pmatrix} = \sum_{i=1}^{1080} \frac{km_{i}}{d_{i}^{3}} \underline{d}_{i}$$
(12)

where

 $k = 6.67 \times 10^{-14} \text{ m}^3/(\text{g}\cdot\text{sec}^2) = \text{the gravitational constant}$   $m_i = 1 \times 10^{19}, \text{ o, or } 1 \times 10^{19}\text{g}$ 

$$\frac{d_{i}}{d_{i}} = \begin{cases}
 d_{x_{i}} \\
 d_{y_{i}} \\
 d_{z_{i}}
\end{cases} = \begin{cases}
 x - x_{i} \\
 y - y_{i} \\
 z - z_{i}
\end{cases}$$

$$\frac{d_{i}}{d_{z_{i}}} = \sqrt{d_{x_{i}}^{2} + d_{y_{i}}^{2} + d_{z_{i}}^{2}}$$
(13)

To minimize the execution time of SUBROUTINE PTMASS, PROGRAM MASPOS was written to compute and save the products of the gravitational constant and each of the 1080 point masses (14) and the earth-fixed x,y,z coordinates of each of the 1080 point masses (15):

$$(km)_{i}$$
,  $i = 1, 2, ..., 1080$  (14)

$$x_{i}, y_{i}, z_{i}, i = 1, 2, ..., 1080$$
 (15)

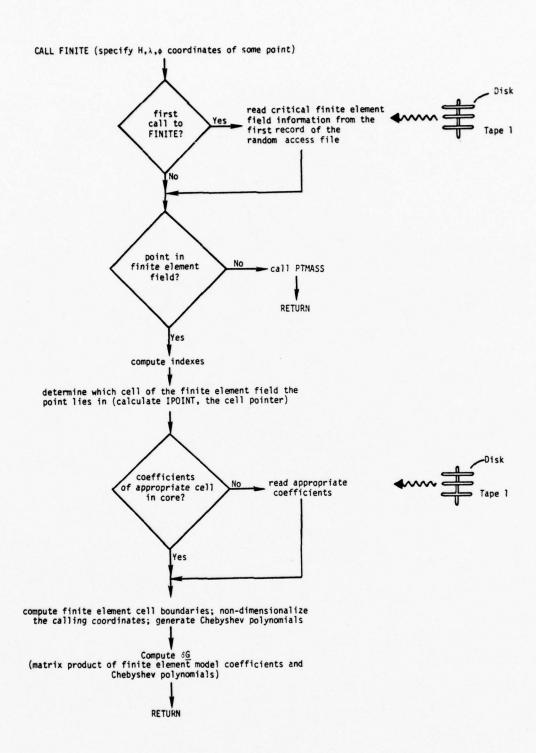


Figure 5 SUBROUTINE FINITE Logic Flow.

#### 6.0 TRADEOFF STUDIES

Several tradeoff studies were conducted on the UVA CDC Cyber 172. The central processor execution times of SUBROUTINE FINITE were compared to SUBROUTINE PTMASS (Model 310). Although it is really a price that is paid only once and a priori in the laboratory, the central processor execution times necessary to create finite element fields were examined. Consideration was given to the tradeoff: total number of finite element coefficients versus NORDER versus cell size, for finite element models of a given region. Lastly, error analyses for a variety of NORDER's and cell sizes were conducted; specifically, the maximum absolute error versus NORDER versus cell size tradeoff was examined.

Figure 6 shows the execution time test results for SUBROUTINE FINITE and SUBROUTINE PTMASS. SUBROUTINE PTMASS has a nearly constant run time. The run time for SUBROUTINE FINITE, however, depends on NORDER, the Chebyshev polynomial order, and on whether or not a random access call is needed. A random access call is needed anytime the set of coefficients currently in core is not the set for the cell in which the calling coordinates lie; that is, the previous calling coordinates were in a different finite element cell than the current calling coordinates.

Observe that the random access time is approximately constant (0.7 ms) regardless of NORDER. This is true because most of the random access time is spent in finding the address of the *first* coefficient on the rotating mass storage device. The read time for one cell's set of coefficients (even NORDER = 6 has only 252 total coefficients for one cell) will be very small compared to the first word address find time. The execution time ratios show that even for the worst cases SUBROUTINE FINITE will be at least 20-50 times faster than an explicit point mass disturbance acceleration model (e.g., SUBROUTINE PTMASS).

A certain price must be paid to generate finite element fields. Execution times involved in running PROGRAM LOCALG and PROGRAM FINEG are shown in Figure 7 for the UVA CDC Cyber 172. The total price for running both programs is also shown. The generation and reduction of the A matrix is a one-time price for any one finite element field. The price

to and the time of the second of the second

SUBROUTINE PTMASS: 123.2 ms

SUBROUTINE FINITE:

NORDER	Execution T No Random Access	imes (ms) Random Access
3	1.7	2.4
4	2.6	3.3
5	3.8	4.5
6	5.4	6.1

(SUBROUTINE FINITE Execution Time) (SUBROUTINE PTMASS Execution Time) = (computational speed advantage)

NORDER	No Random Access	Random Access
3	1/72.5	1/51.3
4	1/47.4	1/37.3
5	1/32.4	1/27.4
6	1/22.8	1/20.2

Figure 6 SUBROUTINE FINITE vs. SUBROUTINE PTMASS
Central Processor Execution Time Comparison.

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		NOI	RDER	
	3	4	<u>5</u>	<u>6</u>
PROGRAM LOCALG				
time to generate A matrix (sec.)	.2	.3	.7	1.4
<pre>time to generate observations   per cell (sec.)</pre>	7.9	15.4	26.6	42.3
PROGRAM FINEG				
time to reduce A matrix (sec.)	.4	2.0	8.7	30.4
<pre>time to calculate coefficients   per cell (sec.)</pre>	.2	.7	1.9	4.3
TOTAL PRICES				
A matrix and reduction (pay once) (sec.)	.6	2.3	9.4	31.8
<pre>observations and coefficients   per cell (sec.)</pre>	8.1	16.1	28.5	46.6

Given:  $H_{cell} = constant = 300 \text{ km}$ 

Figure 7 Central Processor Times to Create Finite Element Fields.

per cell must be multiplied by the total number of cells in the finite element field and added to the price to generate and reduce the A matrix. Note that over 90% of the price paid per cell is just the generation of the gravity observations via SUBROUTINE PTMASS.

The total numbers of coefficients needed for various finite element field models of the 35°×40° region covered by Model 310 are shown in Figure 8. Figure 8 assumes that H<sub>cell</sub> will be constant at 300 km and that 300 km will be the maximum altitude to be modeled. For a given cell size, it shows how many finite element cells would be needed to model the region and how many total coefficients would be needed for various NORDER's to model the region. Based upon error analyses conducted on a small volume near the center of the 35°×40° region, expected maximum error bounds were determined for certain cell sizes and NORDER's. The upper solid and dashed underline indicates an expected maximum absolute error of 1.5 mgals and the lower solid and dashed underline indicates an expected maximum absolute error of 3.0 mgals. Solid underlines in both cases indicate that the expected maximum absolute error was verified for small test volumes while dashes indicate the apparent trend.

In order to study the maximum absolute error as functions of NORDER and cell size, it was decided to consider the maximum absolute error as a function of cell size only for fixed NORDER's (see Figure 9) and to consider absolute error as a function of NORDER for fixed cell sizes (see Figure 10). For all cases, cell altitude (H<sub>cell</sub>) was fixed at 300 km.

Figure 9 shows that the maximum absolute error varies approximately as the square of the cell dimension in  $\lambda$  or  $\phi$ . For constant cell altitude and for small  $\lambda_{cell}$  and  $\phi$ , the volume will vary approximately as the square of  $\lambda_{cell}$  or  $\phi_{cell}$ , if  $\lambda_{cell} = \phi_{cell}$ . Hence the maximum absolute error is really proportional to cell volume. Figure 10 shows that the maximum absolute error varies approximately inversely as the square of NORDER. As the number of coefficients per cell varies approximately as the square of NORDER, the maximum absolute error may be considered to vary nearly inversely with the number of coefficients per cell.

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Cell Size						
$(^{\lambda}_{\text{cell}} \times ^{\phi}_{\text{cell}})$	No. of C	ells	3	4	5	6
1.0°×1.0°	35×40 =	1400	84,000	147,000	235,200	352,800
1.25°×1.25°	28×32 =	896	53,760	94,080	150,528	225,792
1.5°×1.5°	24×27 =	648	38,880	68,040	108,864	163,296
1.75°×1.75°	20×23 =	460	27,600	48,300	77,280	115,920
2.0°×2.0°	18×20 =	360	21,600	37,800	60,480	90,720
2.25°×2.25°	16×18 =	288	17,280	30,240	48,384	72,576
2.5°×2.5°	14×16 =	224	13,440	23,520	37,632	56,448
2.75°×2.75°	13×15 =	195	11,700	20,475	32,760	49,140
3.0°×3.0°	12×14 =	168	10,080	17,640	28,224	42,336

Given:  $H_{cell} = constant = 300 \text{ km}$ 

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- \* Finite element fields above this line are expected to have error max < 1.5 mgals.
- + Finite element fields above this line are expected to have  $\left| \text{error} \right|_{\text{max}} \le 3.0 \text{ mgals.}$

Figure 8 Total Number of Coefficients vs. NORDER vs. Cell Size For a 300 km  $\times$  35°  $\times$  40° Finite Element Field.

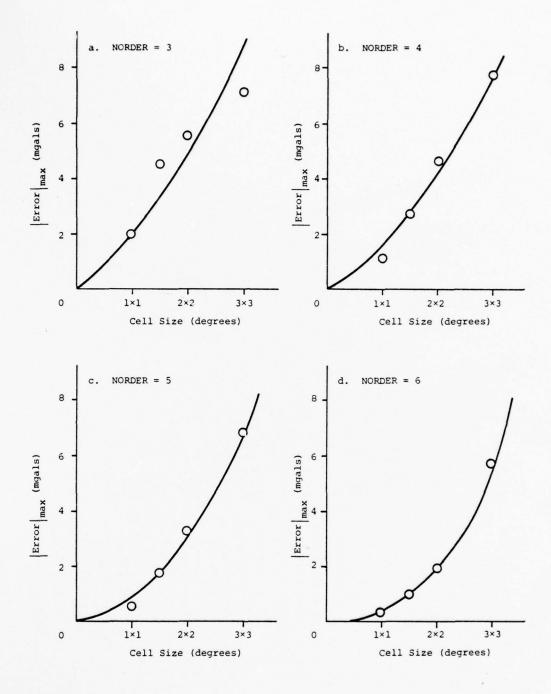


Figure 9 Maximum Absolute Error versus Cell Size for Fixed NORDER's.

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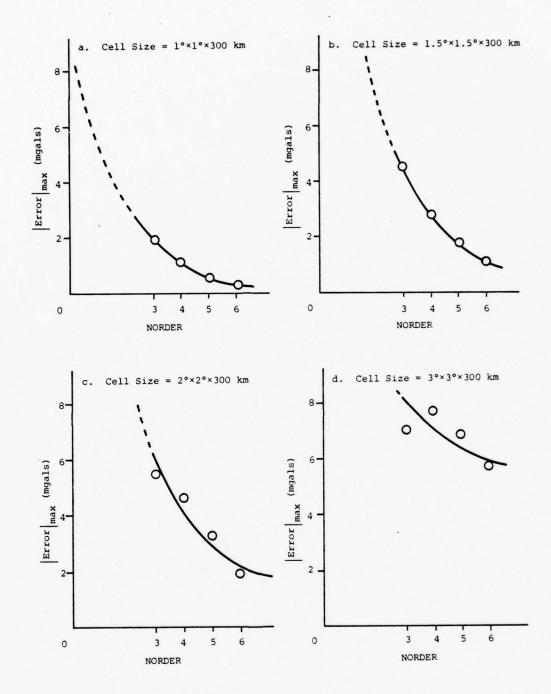


Figure 10 Maximum Absolute Error versus NORDER for Fixed Cell Sizes.

## 7.0 EXAMPLE FINITE ELEMENT GRAVITY MODELS

Based upon the tradeoff studies conducted as described above, two example finite element gravity fields were generated. The 300 km × 10° × 10° region with altitude 0 to 300 km, longitude 70°E to 80°E, and latitude 25°S to 35°S was modeled. A finite element field with NORDER = 3 and with one hundred 300 km × 1° × 1° cells was generated as was a finite element field with NORDER = 5 and the same size and number of cells. An error analysis was conducted on both fields. The finite element model was compared on a point by point basis with Model 310 at sample points "away from the fit points" for altitudes of 1 m, 150 km, and 299 km. The error mean, the RMS error, and the maximum absolute error were determined for the three sets of sample points for both finite element fields and are shown in Figure 11 and Figure 12.

	δg <sub>up</sub>	δgeast	$^{\delta g}$ north
	(mgals)	(mgals)	(mgals)
H = 1 m			
error	005	001	.004
RMS error	.963	.645	.727
error	2.717	2.074	2.236
H = 150 km			
error	023	.003	.009
RMS error	.352	.224	.270
error	1.149	.701	1.039
	, -		
H = 299  km			
error	.018	006	007
RMS error	.424	.277	.322
error	1.130	.771	1.002

Figure 11 NORDER = 3 Example Finite Element Gravity
Disturbance Field.

	δg <sub>up</sub>	$^{\delta g}$ east	$^{\delta g}$ north
	(mgals)	(mgals)	(mgals)
H = 1 m			
error	004	.001	.002
RMS error	.224	.150	.167
error	.918	.637	.581
H = 150  km			
error	.007	002	004
RMS error	.071	.044	.055
error	.216	.157	.215
H = 299  km			
error	002	.001	.001
RMS error	.101	.067	.077
error	.329	.254	.249

Figure 12 NORDER = 5 Example Finite Element Gravity
Disturbance Field.

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#### 8.0 CONCLUSIONS

This effort has demonstrated that:

- Finite element fields are suitable for modeling the gravity disturbance.
- Finite element fields are computationally attractive for modeling the gravity disturbance.

# 9.0 REFERENCES

 Junkins, J. L., "Development of Finite Element Models for the Earth's Gravity Field, Phase I: Macro Gravity Model for Satellite Orbit Integration," University of Virginia, RLES Report No. UVA/525023/ESS77/103, February 1977.

#### APPENDIX - SOFTWARE

A.O FORTRAN IV programs have been developed which create finite element gravity disturbance fields and which evaluate the gravity disturbance within these fields via the methods discussed in this report. Descriptions of these routines and their attendant subroutines are provided here, together with listings of all routines and example executions.

Three major routines were developed - PROGRAM LOCALG, PROGRAM FINEG, and SUBROUTINE FINITE - to create finite element gravity disturbance fields and to evaluate the gravity disturbance within these fields. Two additional routines were developed to provide simulated gravity disturbance data - PROGRAM MASPOS, which, for the 1080 mass points of Model 310, computes and stores the products of each mass point and the gravitational constant together with the geocentric rectangular coordinates of each mass point, and SUBROUTINE PTMASS, which evaluates the gravity disturbance as simulated by Model 310.

- A.1.1 PROGRAM LOCALG generates the A matrix (8) for a "typical" finite element cell and a set of gravity disturbance observations (10) for each cell of a finite element gravity disturbance field. The gravity disturbance observations are evaluations of SUBROUTINE PTMASS on the uniform observation grid in each cell. The user defines a finite element gravity disturbance field by providing the following items as inputs to PROGRAM LOCALG:
  - 1. The integral order of the locally valid Chebyshev polynomial approximating functions, NORDER, (0 < NORDER < 6).
  - 2. The observation grid pattern (same for all cells) to include the number of observations in the up, eastern and northern directions, MOBSU, MOBSE, MOBSN. The total number of observations (MOBSU × MOBSE × MOBSN) should be at least three times the number of coefficients, NC (see Table 1), in the approximating model (3). Furthermore, the number of observations in each direction should be equal (MOBSU = MOBSE = MOBSN). A good rule of thumb is that for all NORDER's between 3 and 6 inclusive, the number of observations in each direction should be one more than NORDER (MOBSU = MOBSE = MOBSN = NORDER + 1).

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The maximum number of observations allowed by the program is 343 (a  $7\times7\times7$  observation grid). Non-uniform sample grids (e.g.  $5\times7\times7$ ,  $6\times5\times4$ , etc.) produce unpredictable results - often an unacceptable fit.

- 3. The size of each finite element cell in H, $\lambda$ , $\phi$  must be specified, HCELL, ALCELL, APCELL. HCELL is cell size in H (meters), (HCELL  $\leq$  [HMAX HMIN]). ALCELL is cell size in  $\lambda$  (degrees), (ALCELL  $\leq$  [ALMAX ALMIN]). APCELL is cell size in  $\phi$  (degrees), (APCELL  $\leq$  [APMAX APMIN]).
- 4. The minimum and maximum  $H, \lambda, \phi$  boundaries of the finite element field, HMIN, ALMIN, APMIN, HMAX, ALMAX, and APMAX. HMIN and HMAX are heights (H) above the reference ellipsoid in meters, (HMIN  $\geq$  0 and HMAX > HMIN). ALMIN and ALMAX are geodetic longitudes ( $\lambda$ ) in degrees, (-180°  $\leq$  ALMIN < ALMAX  $\leq$  180°). APMIN and APMAX are geodetic latitudes ( $\phi$ ) in degrees, (-90°  $\leq$  APMIN < APMAX < 90°).

These thirteen items are input on four data cards. NORDER, MOBSU, MOBSE, and MOBSN are input in (4I10) format on the first card. The remaining three cards must have a (3E20.14) format, with the finite element field delimiters, HCELL, ALCELL, and APCELL on the second card, HMIN, ALMIN, and APMIN on the third card, and HMAX, ALMAX, and APMAX on the last card.

Because gravity disturbance observations are generated within PROGRAM LOCALG by SUBROUTINE PTMASS, PROGRAM LOCALG also requires as input the precomputed products of the gravitational constant and each of the mass points of Model 310 and the precomputed rectangular coordinates of each mass point. These quantities should have previously been stored sequentially via an unformatted write on TAPE 2. PROGRAM LOCALG accesses these quantities with an unformatted read of TAPE 2.

PROGRAM LOCALG produces as output an unformatted sequential file, TAPE 3, which contains the A matrix for a typical finite element cell and a set of gravity disturbance observations for each cell of the user specified finite element field. This file is properly formatted for later use by PROGRAM FINEG. PROGRAM LOCALG requires two supporting subroutines - SUBROUTINE CHEBY and SUBROUTINE PTMASS.

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- A.1.2 SUBROUTINE CHEBY (X, N, TA) returns Chebyshev polynomials in array TA, evaluated at a specified value, X, through the specified order, N.  $TA(1) = T_0(X), TA(2) = T_1(X), TA(3) = T_2(X), etc. TA is a seven element array; hence the specified order is bounded, <math>0 \le N \le 6$
- A.1.3 SUBROUTINE PTMASS produces gravity disturbance "observations",  $\delta g_{up}$ ,  $\delta g_{east}$ , and  $\delta g_{north}$ , from (10) to be used by PROGRAM LOCALG and PROGRAM FINEG for fitting purposes and to be used to test as data against SUBROUTINE FINITE. SUBROUTINE PTMASS produces the gravity disturbance data by using the 1080 point masses of Model 310. SUBROUTINE PTMASS requires that COMMON /MASPOS/ be filled with the precomputed products of the gravitational constant and each point mass and the geocentric rectangular coordinates of each point mass. SUBROUTINE PTMASS interfaces with the calling program via COMMON /XYZ/ through which the rectangular coordinates, x, y, z, of the calling point are received and the components of the gravity disturbance  $\delta g_{x}$ ,  $\delta g_{y}$ , and  $\delta g_{z}$  (DELGX, DELGY, DELGZ) are transmitted. The rectangular components of the gravity disturbance must be transformed to geodetic components  $\delta g_{up}$ ,  $\delta g_{east}$ , and  $\delta g_{north}$ , outside SUBROUTINE PTMASS (i.e., in the calling routine).
- A.2.1 PROGRAM FINEG generates sets of locally valid coefficients C<sub>up</sub>, C<sub>east</sub>, and C<sub>north</sub> (9) for each cell of a finite element field which was previously specified by inputs to PROGRAM LOCALG. The only input required by PROGRAM FINEG is the unformatted sequential file, TAPE 3, previously produced by PROGRAM LOCALG. PROGRAM FINEG produces as output a random access file, TAPE 1, which contains, in effect, a finite element gravity disturbance field. The first record of the random access file contains certain important information about the finite element field (NORDER, boundaries, etc.). In addition to the first record, the random access file consists of one record of coefficients (9) for each cell of the finite element field. Thus the number of records in the random access file will be equal to the number of cells in the finite element field plus 1. PROGRAM FINEG requires one supporting subroutine SUBROUTINE ALSO.
- A.2.2 SUBROUTINE ALSQ (A,Y,B,R2,NN,MM,NA) solves the linear least squares problem, ||AB Y|| = minimum. SUBROUTINE ALSQ requires as input the

coefficient matrix, A, the observation vector which is to be fit, Y, the number of rows used in the A matrix, NN, the number of columns used in the A matrix, MM, and the first dimension of the A matrix, NA. NA must be equal to one plus the maximum number of rows in the A matrix (NA =  $NN_{max} + 1$ ). SUBROUTINE ALSQ returns B, the coefficients of the fit and R2, the sum of the squares of the residuals.

- A.3.1 PROGRAM FINTES is a simple error analysis routine. It compares the gravity disturbance as approximated by SUBROUTINE FINITE with the gravity disturbance as evaluated by SUBROUTINE PTMASS (Model 310). Residuals are printed for each point of a user specified grid, along with the mean of the residuals, the root mean square of the residuals, and the maximum absolute value of the residuals, for each component of the gravity disturbance. The following items are required as input to specify the sample grid:
  - 1. The number of samples in each direction, ISTEPH, TSTEPL, and ISTEPP, (ISTEPH > 0, ISTEPL > 0, and ISTEPP > 0). The total number of sample points on the grid will be the product of these three numbers.
  - 2. The minimum sample grid coordinate values ("the lower corner"), HMIN (meters), ALMIN (degrees), and APMIN (degrees) and the maximum sample grid coordinate values ("the upper corner"), HMAX (meters), ALMAX (degrees), and APMAX (degrees), (0 < HMIN < HMAX, -180° < ALMIN < ALMAX < 180°, -90° < APMIN < APMAX < 90°).

These nine items are input on three data cards. ISTEPH, ISTEPL, and ISTEPP are input on the first data card in a (315) format. HMIN, ALMIN, and APMIN are input on the second card in (F10.0, 2F10.2) format and HMAX, ALMAX, and APMAX are input on the third card in the same format. PROGRAM FINTES requires three supporting subroutines - SUBROUTINE FINITE, SUBROUTINE CHEBY (see discussion of this routine above), and SUBROUTINE PTMASS (also discussed above).

Because PROGRAM FINTES uses SUBROUTINE PTMASS, it also requires as input the unformatted sequential file, TAPE 2, containing the precomputed products of the gravitational constant and each point mass of Model 310 and the precomputed rectangular coordinates of each point mass.

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4.3.2 SUBROUTINE FINITE directly replaces Model 310 in applications involving the gravity disturbance  $\delta g$ . Given the geodetic coordinates, H, ALAM, and APHI  $(H,\lambda,\phi)$ , of a specified point, SUBROUTINE FINITE returns the geodetic components, GU, GE, and GN  $(\delta G_{up}, \delta G_{east}, \delta G_{north})$  of the gravity disturbance approximation,  $\delta G$  (3). SUBROUTINE FINITE determines which set of finite element coefficients to use at the specified point, reads them into core, and computes  $\delta G$ . SUBROUTINE FINITE requires SUBROUTINE CHEBY (described previously) to produce Chebyshev polynomials at the specified point. Whenever the specified point does not lie within the bounds of the finite element field currently accessed, SUBROUTINE FINITE calls SUBROUTINE PTMASS, writes an error message, and sets the error flag ISITIN to zero.

Prior to the first call to SUBROUTINE FINITE, the random access file, Tape 1, must be made available to the calling program, and two flags, IFLAG and ISITIN, must be set to zero and included in a COMMON in both the calling program and SUBROUTINE FINITE. This could be COMMON /HLP/ if necessary (COMMON /HLP/ is the vehicle by which SUBROUTINE FINITE receives the calling coordinates and returns the components of  $\delta G$ ), although COMMON /IMARK/ was used.

The requirements of CDC FORTRAN dictated that the CDC utility subroutines, OPENMS, WRITMS, READMS, and CLOSMS, be used to access the random access file. Furthermore, these CDC utility routines require that the user establish an index array (IMARK, in this case) for the random access file. If an index array is required by a particular version of FORTRAN, the index array, IMARK, should be included in a COMMON in both the calling program and SUBROUTINE FINITE.

A.4 PROGRAM MASPOS creates an unformatted sequential file, TAPE 2, which contains the precomputed products of the gravitational constant and each point mass of Model 310 and the precomputed geocentric rectangular coordinates of each point mass. PROGRAM MASPOS requires as input coded information about Model 310 in the form of 30 cards. Each card represents 36 mass points in a grid row of equal latitude where the mass points are 50 longitude minutes apart. The mass points should be listed on each card by increasing longitude and the cards should be arranged in order

of decreasing latitude. PROGRAM MASPOS reads only the coded multiplication factors for each of the 36 mass points of the grid row, where the coded factors indicate the mass of the respective mass points,  $1 = -1.\times10^{19}$  grams, 2 - 0. grams, and  $3 = +1.\times10^{19}$  grams. The coded factors are located in columns 25-60 of each data card. All other data card columns are ignored (for information only, columns 1-9 indicate south latitude in arc minutes of the grid row and columns 10-18 indicate east longitude in arc minutes of the west-most mass point in the row).

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DATA IZ(1), IZ(2), IZ(3), IZ(4), IZ(5), IZ(6), IZ(6), IZ(7), IZ(8), IZ(9),
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         +1,1,2,1,1,2,3,1,2,1,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,3,4,5,1,5,1,2,3,4,1,
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                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    +IY (82), IY (83), IY (94)/
                                                                                                                                                                                                                                                                                                                                                              +IX(82), IX(83), IX(84)/
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          +IZ(82), IZ(83), IZ(84)/
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The second of th

4 FORMAT(1X, 12110) 5 FORMAT(1X, 6E20.12)

TERMINOL 03 Y

P--SOME POINT IN SPACE

VAPIA BLES

H--HEIGHT ABOVE REF. ELLIPSOID ALONG NORMAL TO PALAM--ANGLE LAMBDA, GEODETIC/GEOCENTRIC LONGITUDE (RADIANS) OF APHI--ANGLE PHI, GEODETIC LATITUDE (RADIANS) OF P X,Y,Z--EARTH-FIXED RECTANGULAR COORDINATES R--MAG. OF VECTOR FROM ORIGIN TO P H, ALAM, AP41 -- GEODETIC COORDINATES PN--EAST-WEST RADIUS OF CURVATURE

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INPUTS

HMAX, AL MAX, APMAX -- MAXIMUM BOUNDS OF FINITE ELEMENT FIELD DESIRED HMIN, ALMIN, APMIN--MINIMUM BOUNDS OF FINITE ELMENT FIELD DESIRED MOSSU, MOBSE, MOBSN--OSSEPVATION GRID PATTERN FOR ONE CELL NOODEP--CROER OF CHEBYSHEV POLYNOMIALS DESIRED HOFIL, ALCTLI, APCTLL -- CELL SIZE IN H, LAMBDA, PHI 

SET CONSTANTS

PHVALS, POSITS DF GR A 0= PI / 180. PI = A CCS (-1.) A5=9 A=5A 82=R8\*R8 (2) OV33

PEAN CHEBYSHEV POLYNOMIALS ORDER, GRID PATTERN, CELL SIZE, AND FINITE FLEMENT FIELD BOUNDS

000

COMPUTE THE NUMBER OF COLUMNS IN THE A MATRIX READ (5,1) NORDER, MOBSU, MOBSE, MOBSN IF (FOF(5) .NE. 0.) GOTO 500 READ (5,2) HOFLL, ALCELL, AFCELL IF (EC<sup>2</sup> (5) .NE. 0.) GOTO 500 MOBSU, MOBSE, MOBSN, MOBS HCFLL, ALCFLL, APCELL HMIN, ALMIN, ALMIN, APPIN JF (EOF(5) .NE. 0.) SOTO 500 IF (FCF(5) .NF. 0.) GOTO 500 HMAX, ALMAX, APMAX READ (5,2) HMIN, ALMIN, APMIN READ (5,2) HMAX, ALMAX, APMAX MO BS = MO BSU \* MOBSE \* MOBSN NOPDEP, NC 00 100 IC=1,NP1 NP1=NCRDER+1 PMUM1=2MU-1 RMFM1=2ME-1 PHNM1=2MN-1 WZITF (6,5) WZITF (6,5) WRITE (6,4) WAITE (6,4) WRITF (6,5) WAITF (5, 3) PMU=MC9SU PMN= MCBSN RME=MCBSF NC2=2\*NC CONTINUE NC 3= 3\*4C NC=NC+N DI+N=N NC=0 0 || 2 100

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WRITE(3) HMIN, HMAX, HCELL, ALMIN, ALMAX, ALCELL, APMIN, APMAX, APCELL, NH, + NLAM, NPHI, NOPDFR, NC, NC2, NC3, MOBSU, MOBSE, MOBSN, MOBS, NCELLS COMPUTE THE NUMBER OF CFLLS IN UP, EASTERN, AND NORTHERN DIRECTIONS AND TOTAL NUMBER OF CFLLS IN FINITE ELEMENT FIELD GENERATE THE CHEBYCHEV POLYNOMIALS FOR A TYPICAL CELL. WRITE FIELD DATA ONTO FIRST RECORD OF SEQUENTIAL FILE COMPUTE THE INCREMENT SIZE WITHIN EACH CELL. CONVERT ANGLES FROM DEGREES TO RADIANS RNL= (ALMAX-ALMIN) /ALCELL + .999 RNP= (APMAX-APMIN) / APCELL + .999 FNH= (HMAX-HMIN) / HCELL + .999 APCFLL=APCFLL \* DF GRAD ALCE LL= ALCELL \* DF GRAD NCELL S=NH\* NLAM\*NPHI AL MIN=ALMIN\*DFGRAD AP MI N=APMI N\*DF GRAD AL MAX=ALMAX\*DEGRAD APMAX=APMAX\*DEGRAD DO 200 IH=1, MOBSU DL AM = AL CELL /RMFM1 DPHI = APCFLL/RMNM1 DH=HCELL/RMUM1 PNCELL=NCELLS HO-NIWH-H NPHI = RNP NL AM=RNL HNGHHN 00 0000 000 000

A-13

WRITE ONE LAYER OF THE A MATRIX ONTO THE SEQUENTIAL FILE WRITE(3) ((A(I,J),I=1,MJ),J=1,NG) CONTINJE A (MJ, II) = TX (NX) \* TY (NY) \* TZ (NZ) GENERATE DATA FOR FACH CELL. CALL CHEBY (X3, NORDER, TZ) CALL CHEBY (X1, NOPDER, TX) CALL CHE BY (X2, NORDER, TY) X3=(APHI-APMIN)/APCELL X2=(ALAM-ALMIN)/ALCELL HHCELL=HMCFLL+HCFLL ALMCFL= ALMIN-ALCFLL DO 150 IP=1, MOBSN 00 150 IL=1, MORSE HMCFLL=HMIN-HCFLL X1=(H-HMIN)/HCFLL DO 300 IHC=1,NH AL AM= AL MIN-DLAM APHI = APMIN-OPHI 00 150 II=1,NC ALAM=ALAM+DLAM APHI = APHI + OPHI FILL THE A MATRIX NX=IX(II) NY=IY(II) (II) ZI=ZN 150 CONTINUE MJ=MJ+1 H= H+ DH 200 000 000

RN=SORT (A2\*COSP\*COSP + B2\*SINP\*SINP) TRANSFORM TO X,Y, 7 COORDINATES NO 300 ILC=1,NLAM ALMCFL=ALMCEL+ALCELL APMCFL= APMCFL+ APCFLL AP MCEL = APMIN-APCFLL DO 250 IP=1, MORSN APHI = APHI + DPHI NO 300 IPC=1,NPHI DO 250 IH=1, MORSU 00 250 IL=1, MOBSE ALAM = ALMCEL - DLAM APHI = APMCEL - DPHI PLAM=ALAM+DLAM SINL = SIN (ALAM) COST = COS (ALAM) COSP=COS (APHI) SINP=SIN(APHI) Y= (RN+H) \*COSP 4 SINH) + SIND H=HMCELL-DH CALL PTMASS ZN=82/3N FN=A2/PN X=Y\*COSL X=Y\*COSL Y=Y\*SINL MJ=MJ+1 H=H+DH 0= FW 000 C C C C

DIRECTION COSINE MATRIX FOR 3-2 ROTATION (LAMBDA,-PHI) FROM X,Y,Z, TO WRITE THE THREE OBSERVATION ARRAYS ONTO THE SEQUENTIAL FILE CN=-SIND\*COSL\*GX - SINP\*SINL\*GY + COSP\*GZ CU=COSP\*COSL\*GX + COSP\*SINL\*GY + SINP\*GZ WRITE (3) (DELGE(I), I=1, MOBS)
WRITE (3) (DELGN(I), I=1, MOBS) WRITE(3) (DFLGU(I), I=1, MO9S) GE=-SINL\*GX + COSL\*GY DETEN (MA)=en DELGN (4J)=GN DELGE (MJ) = GE C23=0 C31=-SINF\*COSL C32=-SINP\*SINL UP, FAST, NORTH C12=COSP\*SINL C11=C0SP\*C0SL 250 CONTINUE CONTINUE C 21=-SI NL C13=SINP C33=C0SP C22=C05L STOP 360 600 

SUPROUTINE CHEBY(X,N,TA) DIMENSION TA(?)

SZR CHEBY RETURNS CHEBYCHEV POLYNOMIALS THROUGH ORDER N. EVALUATED AT X, IN VECTOR TA 6660

IF (N .GT. 0 .AND. N .LT. 7) 60TO 10 WRITE(6,5) N

5 FORMAT(24HDILLFGAL NORDER, NORDER=,16)

STOP

10 CONTINUE XBAR=2.\*X-1.

INTO (-1 .LE. XBAR .LE. 1) (0 .LE. X .LE. 1) MAPS XBAP=2X-1 000

TA (1)=1. TA (2) =XBAR

NP1=N+1 NO 20 I=3,NP1 20 TA(I)=2.\*XBAR\*TA(I-1)-TA(I-2)

RETURN

A-17

/MASPOS/PMVALS(1080), POSITS(1080,3) COMMON /XYZ/X,Y,Z,DFLGX,DELGY,DFLGZ SUBPOUTINE PTMASS COMMON

GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND OFLGZ, USING USING THE POINT MASSES OF MASS MODEL 310.

CONSTANT (6.67F-14) AND THE 1080 POINT MASSES (-1.E19, 0., OR +1.E19) POSITS CONTAINS THE PRECOMPUTED EARTH-FIXED X, Y, Z COOPDINATES OF PMVALS CONTAINS THE PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL THE 1080 POINT MASSES. 000000000

TEPMINOLCGY

X,Y,Z--FARTH-FIXED RECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY DISTURBANCE IS TO BE EVALUATED DELGX, DELGY, DELGZ -- EARTH-FIXED RECTANGULAR COMPONENTS OF THE GRAVITY XI, YI, ZI -- EARTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT MASS FOSITS(I,1)=XI POSITS(I,2)=YI POSITS(1,3)=21 000000000

DELGY=0. DELGX=0. DELG 7=0.

DISTSG= DX\* DX+DY\* DY+DZ\*DZ DO 20 I=1,1080 IF (PMVALS(I)) 10,20,10 OIST=SORT(DISTSQ) 0Y=POSITS(I,2)-Y 72=POSITS(I,3)-7 OX=POSITS(I,1)-X CONTINUE 10

TEMP=FMVALS(I) /DIST/DISTSO DELGX=DELGX+(DX\*TEMP)
DELGY=JELGY+(DY\*TEMP)
DELGZ=DELGZ+(DZ\*TEMP) 20 CONTINUE PETURN FND C

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A-19

PROGRAM FINEG AND SUPPORTING SUBROUTINES

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LOCAL SRAVITY MODEL COEFFICIENT DETERMINATION PROGRAM, SECOND PART DEPARTMENT OF ENGINEERING SCIENCE AND SYSTEMS SCHOOL OF ENGINEERING AND APPLIED SCIENCE PROGRAM FINEG (OUTPUT, TAPE6=OUTPUT, TAPE1, TAPE3) CHARLOTTESVILLE, VA 22901 UNIVERSITY OF VIRGINIA DR. JOHN L. JUNKINS JOHN T. SAUNDERS AND

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THE LEAST SOUARES MATRIX (A MATRIX) AND LOCAL GRAVITY OBSERVATIONS. A PROGRAM WHICH CREATES ONE RANDCM ACCESS FILE CONTAINING THE COFFICIENTS FOR A FINITE ELEMENT GRAVITY REPRESENTATION OF A USER SPECIFIED REGION. THIS PROGRAM RELIES ON PROGRAM LOCALG TO SUPPLY

COMMON /FLOATA/HMIN, HMAX, HCELL, &L MIN, ALMAX, &LCELL, APMIN, APMAX, CU(84), CE(84), CN(84), C(252) + APCELL, NH, NL AM, NPHI, NORDER, NC, NC2, NC3 DIMENSION A(344,85), DELG(343) DIMENSION IMARK (1402) DI MENSION

7 FORMAT(1H0,10X,18HALSO CALLED, TIME=,F10.4) 8 FORMAT(1H0,10X,16HCOFFFICIENTS FOR,15,25H CELLS COMPUTED, DELTAT= +,F10.4,14H AVERAGE TIME=,F10.4) FORMAT(1X, 10E12. 4) XXX=SECOND (CP) c

0

MAX081/344/

0

READ (3) HMIN, HMAX, HCFLL, ALMIN, ALMAX, ALGELL, APMIN, APMAX, APCFLL, NH, +NLAM, NPHI, NORDFR, NC, NC2, NC3, MD8SU, MD8SE, MD8SN, MD8S, NCELLS GRAVITY REDUCE THE LEAST SOUARES MATRIX TO UPPER TRIANGULAR FORM FOR EACH SELL, RFAD ONE SFT OF GRAVITY OBSERVATIONS AND COMPUTE THE COEFFICIENT ARRAY FOR FACH COMPONENT OF GRAY PFAD THE LEAST SOUARES MATRIX FROM THE SEQUENTIAL FILE CALL ALSO1 (A, DELG, CU, SUMSQ, MOBS, NC, MAXOB1) CALL ALSO1(A, DFLG, CE, SUMSO, MOBS, NC, MAX3B1) ALSOLA, DELG, CU, SUMSQ, MOBS, NC, MAXOB1) READ (3) ((A(I, J), I=ISTART, ISTOP), J=1, NC) CALL OPENMS (1, IMARK, 1402,0) READ (3) (DELG(I), I=1, MOBS) PEAD (3) (DELG(I), I=1, MOBS) READ (3) (DELG(I), I=1, MOBS) CALL WRITHS(1, HMIN, 16, 1) ISTOP=ISTART + NLAYER-1 NO 40 ID=1,NCFLLS ML AYFR= MOBSF \* MOBSN IH=1,40ASU WRITE (5,7) TIME ISTART=ISTOP+1 XXY=SECOND (CP) PNCELL=NCELLS TIME = XXY-XXX CONTINUE TPOINT=1 ISTOP=0 01 00 10

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C

A CHECK ONLY, READ C FROM RANDOM ACCESS FILE AND PRINT IT STORE THE THREE COFFICIENT ARRAYS IN ONE ARRAY, C CALL A\_SQ1(A, DFLG, CN, SUMSQ, MOBS, NC, MAXOB1) WRITE C ONTO RANDOM ACCESS FILE CALL WRITHS (1,C, NC3, IPOINT) CALL READMS(1,C,NC3,IPOINT) WRITE (5,8) NCELLS, TIME, TAV CALL CLOSMS(1) (C(1), J=1, NC3) IPOINT=IFOINT+1 TAV=TIME/PNCFLL C(IPNC2) = CN(IC) CONTINUE C(IPNC) = CE(IC) XXX= SECOND (CP) ON 30 IC=1,NC IPNC 2=I PNC+NC TI ME = XXX-XX C(IC) =CO(IC) WZITE (5,6) IPNC=IC+NC WRITE (6,6) CONTINUE STOP SV 30 0 1 000 000 000 000

ALSO IS A FORTRAN IV SUBROUTINE TO SOLVE THE LINEAR LEAST SQUARES PROBLEM NORM(AB - Y) = MIN. ITS CALLING SEQUENCE IS CONTAINS UPON RETURN THE COEFFICIENTS OF THE FITE CONTAINS UPON PETURN THE RESIDUAL SUM OF SQUARES. IS THE NUMBER OF ROWS IN THE LEAST SQUARES MATRIX. IS THE NUMBER OF COLUMNS IN THE LEAST SQUARES MATRIX. IS THE FIRST DIMENSION OF THE ARRAY A. REDUCE THE LEAST SOUARES MATRIX TO UPPER TRIANGULAR FORM IS AN ARRAY CONTAINING THE LEAST SQUARES MATRIX. UPON RETURN THE (M+1)-TH COLUMN CONTAINS THE SUBROUTINE ALSOLA, Y, B, RZ, NN, MM, NA) APPPOXIMATING VECTOR AB. IS THE VECTOR TO BE FIT. CALL ALSO(A,Y, B, R2, N, M, NA) DIMENSION A(NA,1),Y(1),B(1) S==S A (L, L) = A (L, L) + S IF (A (L, L). LT. 0.) SS=SS+A (I,L) \*\*2 0=52 + S\*A(L,L) NO 10 I=L, N DO F 0 L=1, M S= SORT(S2) > and z z z MM1=1-1 N1=N+1 M1 = M+1 SS=0. SS=2S HESE MIM ママニマ 10 000000000000000 OCC

A-24

The summarious of the transfer of the state of the state

IF (L. ED. M) GO TO 50

11=1+1

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                                                                                                                                                                                                                                                                               CALCULATE THE COFFICIENT VECTOR
                                                                DD 4c I=L,N
A(I,J)=A(I,J) - A(I,L)*A(N1,J)
A(N1,L)=-S
                                                                                                                                                                                                                                                         A(I, M1) = A(I, M1) - 0*A(I,L)
                                                                                                                                                                                                             DO 90 I=L,N
PP = FP +A(I,L)*A(I,M1)
                                                                                                                                                                                                                                   0=PP/ (-A(L,L) *A(N1,L))
                    D0 20 I=L,N
PP=PP + A(I,L) *A(I,J)
                                                                                                                                                                                                                                                                                                     R(M) = A(M, M1) / A (N1, M)
                                                                                                                                                                                                                                                                                                              IF (M.Eq. 1) GO TO 130
                                                                                                                                 PEDUCE THE VECTOR Y
                                                                                                                                                                                                                                                                                                                                                                     00 110 I=L1,M
PP=PP-4 (L,I)*A(I)
                                                                                                                                                                                                                                                                                                                            DO 120 LL=1, MM1
                                                                                                                                                                                                                                                                                                                                                                                             A(L) = FP/A(N1,L)
                                                                                                                                                                                                                                              00 100 I=L,N
                                                      00 40 J=L1, M
00 30 J=L1,M
                                           A(N1, J) = PP/D
                                                                                                                                                                   00 80 I=1,N
A(I,M1)=Y(I)
                                                                                                                                                                                         NO 100 L=1, M
                                                                                                                                                        FNTRY ALSO1
                                                                                                                                                                                                                                                                                                                                                             PP=A (L, M1)
                                                                                                 CONT INUE
                                                                                                                                                                                                                                                                                                                                                  L1=L+1
                                                                                                            PETURN
                                                                                                                                                                                                                                                                                                                                       1=M-11
        . 0=da
                                                                                                                                                                                                  .0=cd
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FORMATCING, 10x, 24HRESIDUAL SUM OF SQUARES=, E20.12)
                                                                                                                            PERFORM THE BACK CALCULATIONS
                                                                                                                                                                                                                          A(I, M1) = A(I, M1) - D*A(I, L)
CONTINUE
                                                                                                                                                                                                     0=PP/(-A(L,L)*A(N1,L))
                                                                                                                                                                                          PP=PP+4 (I,L) *4 (I, 11)
                                                   DO 140 I=MP1,N
SS=SS+A(I,M1)**2
                                                                                              WRITE (5, 901) R2
                                                                                                                                                  DO 170 LL=1,4
                                                                                                                                                                                                                 00 160 I=L,N
                                                                                                                                                                                 00 150 I=L,N
                                                                        A(I, M1) = 0.
C CALCULATE 82
                                                                                                                                                            L=M-LL+1
                               SS=0.
                                                                                                                                                                                                                                               PETURN
                                                                                                                                                                      PP=0.
                                                                                   SS=24
                                                                                                                                                                                                                                                           ONU
                                130
                                                                                                                                                                                            150
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                                                                                                        901
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EXAMPLE EXECUTION OF PROGRAM LOCALG AND PROGRAM FINEG

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I						8509E-05	.1816E-04	.6402E-05	
						850	.181	. 640	
						10-3	E-06	E-05	
						-2939E-04	9636E-06	2063E-05	
						•			
						.7849E-05	. 8368E-06	.2708E-04	
						•	•	•	.2340
						.4872E-06	2378E-05	.1441E-05	
						.487	23	26	.2340 AVERAGE TIHE≈
						-05	50-3	-05	VERAGI
						1682E-05	.5418E-05	-,5273E-05	340 A
	017					•		•	• 5
	000E+ 000E+ 000E+					.6475E-05	1790E-05	4536E-06	
	000000		596-0	50E-0	27E-0	9.1	-:1		DELTA
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			.395503006059E-08	.171144648150E-08	.1711718734275-08	.2731E-04	5524E-04	-1520E-04 -1864E-06	1 CELLS COMPUTED, DELTAT=
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		.E0.	SUM	SUM	SUM	5628E-05	1087E-06	.2165E-04 .5068E-06	ENTS
	000E+	ALSQ CALLED, TIME=	RESIDUAL SUM OF SQUARES=	RESIDUAL SUN OF SQUARES=	RESIDUAL SUN OF SQUARES=	56	10	21	COEFFICIENTS FOR
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	3000					3353E-04 6083E-06	.4658E-04	.14936-00 .1044E-04 .1764E-05	
						1 1		• • •	

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PROGRAM FINTES, SUBROUTINE FINITE, AND SUPPORTING SUBROUTINES

The Control of the Co

FINTES (INPUT, OUT PUT, TAPES=INPUT, TAPE6=OUTPUT, TAPE1, TAPE2) FORMAT(1H0,4X,1HH,8X,6HLAM9DA,5X,3 +PHI,7 X,5HERRGU,5X,5HERRGE,5X + SHEPRGN, 5X, 4HPMGU, 6X, 4HPMGE, 6X, 4HPMGN, 6X, 4HFIGU, 6X, 4HFIGE, 6X, \*\*\*\*\* FORMAT(1X, 26HARS. VALUES OF MAX. ERRORS, 4X, 3(1X, 5PF9.4))
FORMAT(46H0\*\*\*\*\* NEGATIVE OR ZERC NO. OF STEPS \*\*\*\*\* FORMAT(4X,4H(KM),1X,2(5X,5H(DEG)),2X,9(3X,7H(MGALS))) FORMAT(1X, 19HSTANDARD DEVIATIONS, 11X, 3(1X, 5PF9.4)) FORMAT(1H0,10x,22HFINITE TIME/EXECUTION=,F9.4) FORMAT(1H0,10x,22HPTMASS TIME/EXECUTION=,F9.4) FJRMAT(1X,-3PF10.3,2(0PF10.4),9(1X,5PF9.4)) /MASPOS/PMVALS(1080), POSITS(1080,3) FORMAT(///, 1X, 5HMEANS, 25X, 3(1X, 5PF9.4)) /IMARK/IFLAG, ISITIN, IMARK (1402) /HLP/H, ALAM, APHI, GU, GE, GN FORMAT(2x, 9(1H-), 11(2x, 8(1H-))) DEAD(5,1) ISTEPH, ISTEPL, ISTEPP /FLOATA/REA (9), INT (7) /XYZ/X,Y,Z,GX,GY,GZ CALL CPENMS (1, IMARK, 1402,0) CALL READMS(1, REA, 16, 1) FORMAT(1X, F10.0, 7F10.2) READ (2) PMVALS, POSITS 3/6356774.504/ FORMAT(F10.0,7F10.2) .NE. 0.) DATA A/6378160./ FORMAT(1X, 12110) DF GP A D= PI / 180. PI = A CCS (-1.) FORMAT(1615) FORMAT (1H1) IF (ECF (5) ISITIN=0 PROGRAM + CHFIGN) NOMMOU NOWWOO COMMON 6 +8= 2d COMMON COMMON A + A = 5A DATA 11 12 01 20 t

SAMMEN

WRITE (6, 4) ISTEPH, ISTEPL, ISTEPP READ(5,2) HMIN, ALMIN, APMIN IF (EOF(5) .NE. 0.) GOTO 40 READ(5,2) HMAX,ALMAX,APMAX IF (EDF(5) .NE. 0.) GOTO 40 WRITF (6,5) HMIN, ALMIN, APMIN WRITE (6,5) HMAX, ALMAX, APMAX DL AM = (AL MAX-AL MIN) /RLM1 DPHI = (APMA X-APMIN) / RPM1 IF (IHM1) 21,22,23 21,25,26 21,28,29 DH=(HMAX-HMIN)/RHM1 ILM1=ISTEPL-1 IPM1=ISTEPP-1 IHM1 = ISTEPH-1 HZITF (6, 15) WRITE (6,3) WRITE (6,6) WRITF (6,7) IF (ILM1) IF (IFM1) WAITE (6,8) PHM1 = IHM1 PLM1=ILM1 PPM1 = IPM1 FTIME=0. PTIME=0. GOTO 24 DLAM= 0. DPHI = 0. GOTO 30 GOTO 40 G0T0 27 FGU=0. E35=0. 0 H = 0 27 59 23 25 56 30 21 22

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RN=SORT (A2\*COSP\*COSP + B2\*SINP\*SINP) FTIME=FTIME+TAFT-TBEF 00 35 I=1, ISTEPH 00 35 K=1,ISTEPP 00 35 J=1, ISTFPL TAFT = SECOND (CP) TAFF = SECOND (CP) TBEF = SECOND (CP) ALAM=AL \* DE GRAD APHI = AP \* DE GRAD COSL = COS (ALAM) SINL = SIN (ALAM) COSP=COS (APHI) SINP=SIN(APHI) AL = ALMIN-DLAM J= (ZN+H) \*SINP AP=APMIN-DPHI Y= (RN+H) +COSP CALL FINITE AL=AL+JLAM AP=AP+DPHI HO-NIWH=H FGFMAX=0. EGUMAX=0. FGNMAX=0. PN=A2/2N 7N=82/3N SDEGE=0. SDEGN=0. X=Y\* COSL Y=Y\*SINL SDEGU=0. HO+H=H GUF= GU GEF= GE GNF=GN

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WRITE (6,11) H, AL, AP, FRRGU, ERRSE, FRRGN, GU, GE, GN, GUF, GFF, GNF LN=+SINP\*COSL\*6X - SINP\*SINL\*6Y + COSP\*GZ CU=COSP\*COSL\*GX + COSP\*SINL\*GY + SINP\*GZ EGNMAX=AERRGN EGUMA X=AFRRGU EGEMAX=AERRGE WAITE (5,13) SDEGU, SDEGE, SDEGN WAITE (5,14) EGUMAX, EGEMAX, SGNMAX IPTS=ISTEPH\*ISTEPL\*ISTEPP SDEGE = SORT (SPEGE /RIPTM1) SDEGU-SORT (SDEGU/RIPTM1) SDEGN=SORT (SDEGN/RIPTM1) WRITE (6, 12) EGU, EGE, FGN SOFGU-SOFGU+ERRGU SOFGE-SOFGE+ERRGE SOFGN-SOFGN+FRRGN GE=-SINL\*GX + COSL\*GY PTIME =PTIME + TAFT - TREF IF (AFREGU.GT. EGUMAX) IF (AFRAGE, GT. EGEMAX) TF (AERRGN. GT. EGNMAX) FT IME = FT IME / RIPTS AERE GN= ABS (EPRGN) AERRGU= ABS (FRRGU) AE PRGE= ABS (ERRGF) TAFT = SE COND (CP) RIPTM1=RIPTS-1. FGN= FGN+TRRGN FGU=FGU/RIPTS EGN=F GV /RIPTS FGU=EGU+ERRGU FGE=FG: + ERRGE ESF=EGE/RIPTS FRPGU=GU-GUF FRRGE = 3 E - G EF ERREN-GNF FIMASS PIPTS=IPTS CONTINUE 35

PTIME=PTIME/RIPTS
WRITE(5,9) FTIME
WRITE(6,10) PTIME
GOTO 20
GOTO 20
CALL C\_OSMS(1)
STOP
FND

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EVALUATES THE GRAVITY ANOMALY, DELTA G, FROM /FLDATA/HMIN, HMAX, HCELL, ALMIN, ALMAX, ALCELL, APMIN, APMAX, +IX(10), IX(11), IX(12), IX(13), IX(14), IX(15), IX(16), IX(11), IX(118), + TX (19), IX (20), IX (21), IX (22), IX (23), IX (24), IX (25), IX (26), IX (27), +IX(28), IX(29), IX(30), IX(31), IX(32), IX(33), IX(34), IX(35), IX(36), +IX(37),IX(38),IX(39),IX(40),IX(41),IX(42),IX(43),IX(44),IX(45), +IX(46), IX(47), IX(48), IX(49), IX(50), IX(51), IX(52), IX(53), IX(54), A SUPRDUTINE WHICH EVALUATES THE GRAVITY ANOMALY, DELTA G, FRANY ONE RANDOM ACCESS FILE CREATED BY PROGRAMS LOCALG AND FINEG. DATA IX(1), IX(2), IX(3), IX(4), IX(5), IX(6), IX(7), IX(8), IX(9), FINITE ELEMENT GRAVITY FIELD GRAVITY EVALUATION SUBROUTINE DEPARTMENT OF ENGINEERING SCIENCE AND SYSTEMS SCHOOL OF ENGINEERING AND APPLIED SCIENCE /IMARK/IFLAG, ISITIN, IMARK(1402) +APCELL, NH, NLAM, NPHI, NORDER, NC, NC2, NC3 TX(7), TY(7), TZ(7), C (252) /HLP/H, ALAM, APHI, GU, GE, GN 22901 DI MENSION IX (84), IY (84), IZ (84) UNIVERSITY OF VIPGINIA /XYZ/X,Y,7,6X,6Y,6Z DR. JOHN L. JUNKINS CHARLOTTESVILLE, VA JOHN T. SAUNDERS SURPOUTINE FINITE AND DIMENSION COMMOD COMMON COMMON COMMON 0000000

+IX(55), IX(56), IX(57), IX(58), IX(59), IX(60), IX(61), IX(62), IX(63), +IX(64), IX(65), IX(56), IX(67), IX(68), IX(69), IX(70), IX(71), IX(72),

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+IY (10), IY(11), IY (12), IY (13), IY (14), IY (15), IY (16), IY (17), IY (18), +IY(19), IY(20), IY(21), IY(22), IY(23), IY(24), IY(25), IY(26), IY(27), +IY(28),IY(29),IY(30),IY(31),IY(32),IY(33),IY(34),IY(35),IY(36), +IY (37), IY(38), IY (39), IY (40), IY (41), IY (42), IY (43), IY (44), IY (45), +IY (46), IY (47), IY (48), IY (49), IY (50), IY (51), IY (52), IY (53), IY (54), +IY(55),IY(56),IY(57),IY(58),IY(59),IY(50),IY(61),IY(62),IY(63), +IY(64), IY(65), IY(66), IY(67), IY(68), IY(69), IY(70), IY(71), IY(72), +TZ(10), IZ(11), IZ(12), IZ(12), IZ(13), IZ(14), IZ(15), IZ(16), IZ(17), IZ(18), +12(19),12(20),17(21),12(22),12(23),12(24),12(25),12(26),12(27), +12(28),12(29),17(30),12(31),12(32),12(33),12(34),12(35),12(36), +IZ(37),IZ(38),IZ(39),IZ(40),IZ(41),IZ(42),IZ(43),IZ(44),IZ(45), +IZ(46),IZ(47),IZ(48),IZ(49),IZ(50),IZ(51),IZ(52),IZ(53),IZ(54), +IZ(55),IZ(56),IZ(57),IZ(58),IZ(59),IZ(60),IZ(61),IZ(62),IZ(63), +12(64),12(65),12(66),12(67),12(68),12(69),12(70),12(71),12(72), .IX(73), IX(74), IX(75), IX(76), IX(77), IX(78), IX(79), IX(80), IX(81), +IY(73),IY(74),IY(75),IY(76),IY(77),IY(78),IY(79),IY(80),IY(81), +12(73),12(74),17(75),12(76),12(77),12(78),12(79),12(80),12(81) +1,1,2,1,1,2,3,1,2,1,1,2,3,4,1,2,3,1,2,1,1,2,1,1,2,3,4,5,1,2,3,4,1, +2,3,1,2,1,2,3,4,2,3,4,5,6,1,2,3,4,5,1,2,3,4,1,2,3,4,1,2,3,1,2,1,1,2,3,4, DATA IZ(1), IZ(2), IZ(3), IZ(4), IZ(5), IZ(6), IZ(7), IZ(8), IZ(9), +3, 3, 4,4,5,101,11,11,11,1,1,2,2,2,2,3,3,3,3,3,4,4,4,5,5,5,6,1,1,1,1,1 DATA IY(1), IY(2), IY(3), IY(4), IY(5), IY(6), IY(7), IY(8), IY(9), +1,2,1,1,3,2,1,2,1,1,4,3,2,1,3,2,1,2,1,1,5,4,3,2,1,4,3,2,1,4,3, +2,1,2,1,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,3,2,1,2,1,1,7,6,5,4, +1,1,1,2,1,1,1,1,2,2,3,1,1,1,1,1,2,2,2,3,3,4,1,1,1,1,1,1,1,2,2,2,3, +1,1,1,1,2,2,2,2,2,2,3,3,3,3,3,4,4,4,4,4,5,5,5,5,6,6,7/ +3,2,1,6,5,4,3,2,1,5,4,3,2,1,4,3,2,1,3,2,1,3,2,1,2,1,1/ +IY (82), IY (83), IY (84)/ +IX (82), IX(83), IX (84)/ + TZ (82), IZ(83), IZ(84)/

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COMPARE COORDINATES TO REGION BOUNDARIES. IF POINT IS OUTSIDE FINITE

FLEMENT FIFTO, PRINT FRFOR MESSAGE AND CALL GRAV

100 IF (H.LT. HMIN.OR. H. GT. HMAX. OR. ALAM.LT. ALMIN.OR. ALAM.GT. ALMAX.OR.

+APHI.LT.APMIN.OR.APHI.GT.APMAX) GOTO 400 ISITIN=1

COMPUTE VARIOUS INDEXES TO HELP FIND THE RIGHT CELL IN THE REGION 400

PI=I PJ=J

RKIK

COMPUTE IPDINT, THE POINTER FOR THE SET OF COEFFICIENTS AT THESE COOPDINATES 0000

IPOINT=(I-1)\*NPHI\*NLAM + (J-1)\*NPHI + K + IF (IPOINT .EO. IFLAG) GOTO 200

PEAD A NEW SET OF COEFFICIENTS ONLY IF PROPER SET IS NOT IN CORE 000

CALL READMS(1,C,NC3,IPOINT) IFLAG=IPOINT C C FIND THE MINIMUM CFLL BOUNDARY COORDINATES C

200 HMCELL=HMIN + (RI-1.)\*HCELL ALMCFL=ALMIN + (RJ-1.)\*ALCELL APMCEL=APMIN + (RK-1.)\*APGGLL

C C NON DIMENSIONALIZF THE CALLING COORDINATES C

X1=(H-HMCELL)/HCFLL X2=(ALAM-ALMCEL)/ALCFLL

GENERATE CHEBYCHEV POLYNOMIALS AT NON-DIMENSIONALIZED COORDINATES FVALUATE DELTA G-UP, DELTA G-EAST, AND DELTA G-NORTH CN=-SINP\*COSL\*GX - SINP\*SINL\*GY + COSP\*GZ GU=COSP\*COSL\*GX + COSP\*SINL\*GY + SINP\*GZ AAA=TX(NX) \*TY (NY) \*T Z (NZ) CHEBY (X1, NORDER, TX) CHEBY (X2, NORDER, TY) CHEBY (X3, NORDER, TZ) GE =- SINL\*GX + COSL\*GY GN=GN + AAA\*C(IIFNC2) GE=GF + AAA\*C(IIPNC) GU=GU + AAA\*C(II) 00 300 II=1,NC COSL = COS (ALAM) SINL=SIN(ALAM) COSP=COS (APHI) SINP=SIN(APHI) II PNC 2= II+NC2 IIPNC=II+NC CALL PTMASS (II) XI=XN NY=IY(II) (11) 21=211 CONTINUE CONTINUE FETUPN 6U=0. GE = 0 . SN=0. CALL CALL 005 300 000 ccc

X3 = (APHI-APMCFL) /APCFLL

C

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FORMAT(49H POINT NOT IN FINITE ELEMENT FIELD, PTMASS CALLED) RETURN FND

The second state of the second second

(-1 .LE. XBAR .LE. 1) SZR CHEBY RETURNS CHEBYCHEV POLYNOMIALS THROUGH OPDER N, EVALUATED AT X, IN VECTOR TA FORMAT(24H0ILLFGAL NORDER, NORDER=, 16) INTO IF (N .GT. 0 .AND. N .LT. 7) GOTO 10 WRITF(6,5) N (0 .LE. X .LE. 1) DO 20 I=3, NF1
TA(I)=2, \*XBAR\*TA(I-1)-TA(I-2) SUPROUTINE CHEBY(X,N,TA) DIMENSION TA(7) XBAP=2X-1 MAPS XBAR=2. \*X-1. TA (1)=1. TA (2)=XBAR 10 CONTINUE NP 1=N+1 FND STOP 50 u

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/MASPOS/PMVALS(1080), POSITS(1080,3) COMMON /XYZ/X,Y,Z,OFLGX,OFLGY, DFLGZ SUBPOUTINE PTHASS COMMON

GIVEN THE RECTANGULAR COORDINATES, X, Y, Z, OF A POINT, THIS ROUTINE RETURNS THE COMPONENTS OF THE GRAVITY DISTURBANCE, DELGX, DELGY, AND DELGZ, USING USING THE POINT MASSES OF MASS MODEL 310.

PMVALS CCNTAINS THE PRECOMPUTED PRODUCTS OF THE GRAVITATIONAL CONSTANT (6.67E-14) AND THE 1080 POINT MASSES (-1.E19, 0., OR +1.E19) POSITS CONTAINS THE PRECOMPUTED EARTH-FIXED X, Y, Z COORDINATES OF THE 1080 POINT MASSES.

TERMINOLOGY

DELGX, DELGY, DELGZ -- FARTH-FIXFD RECTANGULAR COMPONENTS OF THE GRAVITY x,Y,Z--EARTH-FIXFD PECTANGULAR COORDINATES OF POINT AT WHICH GRAVITY XI, YI, ZI--EAPTH-FIXED RECTANGULAR COORDINATES OF THE ITH POINT DISTURBANCE IS TO BE EVALUATED POSITS(I,1)=XI POSITS(I,2)=YI POSITS(1,3)=71 

DELGY=0. DELGX=0. DELG 7=0.

C

ZU\*ZO+AU\*AU\*XO\*XO=DSISIO IF (PMVALS(I)) 10,20,10 DIST = SORT (DISTSO) NY=POSITS(I,2)-Y DZ=POSITS(I, 3)-Z 0X=POSITS(1,1)-X 00 20 I=1, 1080 CONTINUE 10

TEMP=FMVALS(I)/DIST/DISTSO
DELGX=DELGX+(DX\*TEMP)
DELGZ=DELGZ+(DZ\*TEMP)
20 CONTINUE
RETURN
FND

0

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EXAMPLE EXECUTION OF PROGRAM FINTES

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	I C N	20	1117	. 933	4 4	.338	. 872	4110	070	617	. 918	.241	. 141	108	226	.380	.158	.572	780	767		514	696	.133	.277	699	926	. 875	181	476	020	. 806	004.	331	2620	808	408	. 425	. 654	. 507	121	233	516	623	424	8	
	IS:	20	2.755	2.099	1.051 2.879	3.264	3.277	2.943	2626	3.179	3.420	3.381	3.083	2.772	3.214	3.443	3.481	1.205	2.070	2.687	3.507	5.460	.608	.417	606.	601	804	.770	184.	876.	767	.863	.778	. 535	.187	726	811	. 807	.758	.105	.384	.617	070	339	261	996	
	FIGJ (MGALS)	96	3.521	3.475	445	5.446	6.330	6.189	5.959	8.087	9.030	8.893	1.169	10.527	1.628	11.591	10.512	10.133	2.174	14.129	142.41	612	1.747	2.220	.125	2.012	3.134	3.537	3.317	2 . 63	4.358	4.789	0.5.4	3.703	4.038	C 40	5.801	116.4	4.675	6.341	7.120	7.106	1.2391	1.631	763	.726	
	GAL	7.2174	.19	.91	<b>9</b> ~	. 98	.50	99	2 -	64	2.37	• 32	3.41	2.0	82	16.	3.07	.64	• 76	. 55		.22	. 80	• 56	999		9 8	.24	26	71.	8 8	.21	.40	56	•		103	.39	50	06.	• 16	434	t d	6 2	3 6	20	
	PHGE	13	3.998	2.328	3.389	4.235	4.226	3.220	1.001	3.178	3.574	3.594	3.410	1.350	2.857	3.630	4.480	0.748	1.416	2.159	3.161	4.823	.909	.913	.826	178	247	.246	.166	110.	450	445	.377	.246	474	201	439	.328	.475	454.	.413	345	017	037	029	987	
	10 1	: 3	3.453	3.268	324	6.804	7.710	6.324	5.645	9.991	0.978	9.223	90000	11.974	13.297	1.985	-9.405	-9.364	13.558	5.379	13.268	826	1.349	1.762	1 10	1.741	2.297	2.720	2.981	3.085	3.283	3.723	3.930	490.4	3.647	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7	5.001	5.085	4.579	5.225	5.712	6.003	1.020	1.204	1.355	1.465	
	ERRGN (MGALS)	00	30	7	2.5	8	.365	. 104	2 2	.873	. 545	59	1.272	600	402	.325	.915	.073	626	203	260	86	.166	.136	70	366	8 2	.366	4.3	. O . O	3 5	.411	.881	20	4618	200	17	.035	53	.600	15	9 9	000	003	15	.109	
	ERRGE (MGALS)	.60	245	.228	100	971	948	.277	252	.001	153	213	. 327	2 4 5	356	.187	968	.536	959	. 528	805	.636	669	.503	.082	A B G	.557	523	.321	.033	316	4117	004.	.288	. 300	200	371	6440	717	348	.028	.272	165	301	231	.021	
	ERRGU (MGALS)	39	068	207	30	1.358	380	134	410	906	948	329	40/	1.447	699	393	106	169	784	249	200	214	397	125	180	270	836	817	335	176	075	990	828	361	392	240	199	108	960	116	104	102	217	427	407	260	
-29.90 -29.10	PHI (OEG)	. 900	29.500	29.300	29.900	29.700	29.500	29.300	29. 900	29.700	29.500	29.300	001.62	29.700	29.500	29.300	29.100	29.900	29.700	29.500	29.100	29.900	29.700	29.500	29.300	29.900	29.700	29.500	29,300	29. 900	29.700	29.500	29.300	29.100	29.700	29.500	29.300	29.100	29.900	29.700	29.500	29.300	29.900	29.700	29.500	29.300	
5 75.10 75.90	LAMBDA (DEG)	5.1	5.100	5.100	5.100	5.300	5.300	5.300	5.500	5.500	5.500	5.500	004.4	5.700	5.700	5.700	5.700	2.900	5.900	5.900	5.900	5.100	5.100	5.100	5.100	5.300	5.300	5.300	5.300	5.500	5.500	5.500	2.500	5.500	5.700	2.700	5.700	5.700	2.900	2.900	2.900	5.900	5.100	5.100	5.100	5.100	
4 270001.	E SH	.001	00	00	00	3	00	00	3 3	00	00	00	00	3 5	000	00	0.0	00	00	90	3 3	00.0	0.00	0.00	90.001	00.00	00.0	00.0	00.00	000	00 -1	0.00	00.0	3.00	00.0	000	00.0	00.0	00.0	00.0	00.0	00.00	80.00	00.0	80.00	80.00	

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100.	20100	1.67	2400.	•		6 26 3	210	200	2000		14010
100.08	5.300	9.900	2886	•	v	*	110	010	1.062	07.	0 3 0 3
80.001	300	29.700	2066.	•	<b>3</b> 0	170.	261.	070	0/11.0	? :	2050
80.001	300	29.500	****	2381	<b>-</b>	-	1010	200	2022	. 41	45620
100	000	0000	131.70	•	*****	700	6130	0.5	1 686	90	7917
700.00	0000	001 -62	61470-	•	э,	700	100.	200	0000		1003
180.001	75.5000	-29.7000	5334	- 1168	0760	-2.0710	2.3030	6629	-2.6044	2.4177	6058
80.001	500	29.500	.4116		-	218	.283	319	2.630	. 43	.1694
80.001	500	29.300	0585		~	315	235	0 19	2.374	.36	2788
80.001	500	29.100	4265			357	161	378	1.930	.24	4676
.001	700	900	. 0297		0	337	.363	922	2.366	. 25	.8306
89.001	700	29.700	3315	.0486		524	.361	624	2.916	.31	.6218
80.001	700	29.500	.3665		-	668	.334	292	3.035	. 33	.1435
80.001	5.700	9.300	.0575	0618	10	.758	.282	690	2.816	. 34	3335
80.001	5.700	29.100	4360		0	.789	.208	451	2.353	.36	5393
30.001	5.900	29.900	4722		-	795	.379	929	2.323	.17	. 8286
0.001	5.900	29.700	.1401	.2153	0	979	.365	611	3.119	.15	.6162
0.001	5.900	29.500	. 3114	.1759	-	1117	.330	259	3.429	.15	.1472
0.001	5.900	29,300	.1443	•	-	199	.274	122	3.344	.21	3076
1.001	5.900	100	2594	1488		.218	.198	523	.959	. 34	4771
100.0	5.100	29.900	.2685	•	0	446	0.1	333	1.212	68	.3806
10000	5.100	29.700	0437		.2	.014	20	196	.971	.92	. 4342
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PROGRAM MASPOS

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PROGRAM MASPOS (INPUT, OUTPUT, TAPE5=INPUT, TAPE6=OUTPUT, TAPE2)
DIMENSION MASSES (1080), PMVALS (1080), POSITS (1080,3) FORMAT(1H1,3X,9HMASSES(I),18X,1HX,19X,1HY,19X,1HZ) READ(5,903) (MASSES(K), K=ISTART, ISTOP) KNI=A2/SQRT (A2\*COSP\*COSP+B2\*SINP\*SINP) A,8/6378160.,6356774.504/ FORKAT (1X, 16, 4X, 3 (1PE 20.10)) FORMAT (1X, 4 (3X, 17 (1H-1)) FORMAT(1X, 4(1PE20.10)) FORMAT(24X, 3611, 20X) BIGG/6.67E-14/ RAUMIN=10800./PI ISTART=ISTART+36 ISTOP=ISTART+35 PHI=PHIN/RADMIN PHIM=PHIM-50. SINP=SIN (PHI) COSP=COS (PHI) PI=ACOS (-1.) 00 10 I=1,30 00 20 I=1,30 ALAMBM=3575. 00 20 J=1,36 RHI=-80000. ISTART=-35 PHIM=-875. CONTINUE A2=A\*A 82=8\*8 IPT=0 DATA 903 | 901 902 10 C

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WRITE (6,905) PMVALS (IPT), POSITS (IPT, 1), POSITS (IPT, 2), POSITS (IPT, 3) POSITS(IPT, 3) = (82\*RNI/A2+RHI) \*SINP PMVALS(I)=RMI\*BIGG\*1.E19 POSITS(IPT,2)=TEMP\*SINL POSITS (IPT, 1) = TEMP \* COSL WRITE(2) PMVALS, POSITS TEMP= (RNI+RHI) \*COSP ALAMB=ALAMBM/RADMIN ALAMBM=ALAMBM+50. SINL=SIN (ALAMB) COSL=COS (ALAMB) 00 30 I=1,1080 MI=MASSES(I)-2 I=1,20 J=1,54 WRITE (6, 901) WRITE(6,902) IPT=IPT+1 IPI=IPI+1 CONTINUE CONTINUE CONTINUE RMI=MI 04 00 07 00 IPI=0 STOP END 20 30 0 4 C S S S C

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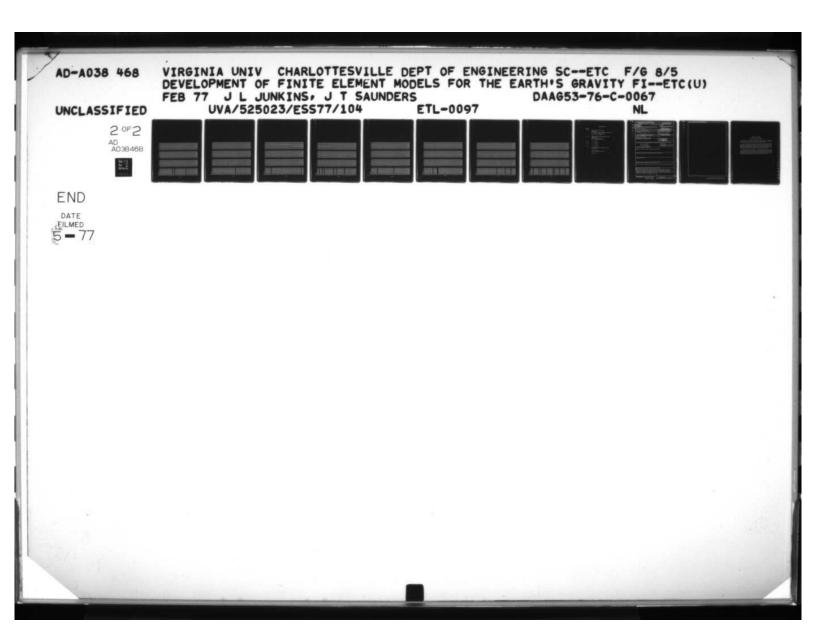
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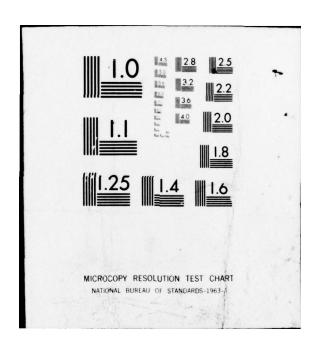
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## UNIVERSITY OF VIRGINIA

## School of Engineering and Applied Science

The University of Virginia's School of Engineering and Applied Science has an undergraduate enrollment of approximately 1,000 students with a graduate enrollment of 350. There are approximately 120 faculty members, a majority of whom conduct research in addition to teaching.

Research is an integral part of the educational program and interests parallel academic specialties. These range from the classical engineering departments of Chemical, Civil, Electrical, and Mechanical to departments of Biomedical Engineering, Engineering Science and Systems, Materials Science, Nuclear Engineering, and Applied Mathematics and Computer Science. In addition to these departments, there are interdepartmental groups in the areas of Automatic Controls and Applied Mechanics. All departments offer the doctorate; the Biomedical and Materials Science Departments grant only graduate degrees.

The School of Engineering and Applied Science is an integral part of the University (approximately 1,400 full-time faculty with a total enrollment of about 14,000 full-time students), which also has professional schools of Architecture, Law, Medicine, Commerce, and Business Administration. In addition, the College of Arts and Sciences houses departments of Mathematics, Physics, Chemistry and others relevant to the engineering research program. This University community provides opportunities for interdisciplinary work in pursuit of the basic goals of education, research, and public service.